New illustration for Mixed Mode Fracture Mechanics Analysis of Central-Crack Plates Using Crack Extension Technique and Matlab

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Abstract: New illustration for mixed mode fracture mechanics analysis of central cracked plates using crack extension technique and Matlab Environment is presented. The technique of crack extension is applied to the computation of mixed mode stress intensity factors in linear elastic fracture mechanics for these plates for different loads. The technique uses the Brown approximate solutions for stress intensity factors and the Westergaard analytical solutions for stress and displacement near a crack tip in finite plate to calculate crack extension during each load step using an

proved to be a good tool for computation and results illustration for mixed mode stress intensity factors. The results were illustrated in a new form which is convenient for engineers and fracture mechanics analyst. The developed procedure reduced the need for sophisticated numerical analyses, which require more time and effort, to calculate the same parameters tackled in this research.

ايضاح جديد لتحليل الطور المختلط لميكانيكية الكسر لصفائح متمركزة الشق باستخدام تقنية تمدد الشق والماتلاب

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الخلاصة: رؤية جديدة لتحليل الطور المختلط لميكانيكية الكسر لصفائح متمركزة الشق باستخدام تقنية تمدد الشق طبقت لحساب معاملات تركيز الاجهاد للطور المختلط لميكانيلية الكسر الخطية لهذه الصفائح تحت احمال شد مختلفة. التقنية استخدمت حلول براون التقريبية لمعاملات تركيز الاجهاد وكذلك حلول وستركارد التحليلية للاجهاد والازاحات بالقرب من راس الشق في صفيحة محددة الطول لحساب تمدد الشق خلال كل خطوة حمل باستخدام تقنية الزيادة. برنامج ماتلاب طور لهذا الغرض واثبت انه اداة جيدة للحسابات وعرض لنتائج لمامعلات تركيز الاجهاد للطور المختلط. النتائج عرضت بشكل مفيد للمهندسين ومحللي ميكانيكية الكسر. الطريقة المطورة تقلل الحاجة الى طرق عددية معقدة والتي تحتاج الى وقت وجهد اكثر للحصول على نفس النتائج التي عالجها هذا البحث.

1-Introduction

A diversity of techniques are currently available for calculating mixed mode stress intensity factors for linear elastic fracture mechanics problems. The main aim of these techniques is the ability to predict the size of crack which will propagate under a mixed loading in a given material, from measurement of the size of the crack under another loading condition in the same material, the relationship between these sizes is dependent on the conditions near the tip of the crack. Provided that such a region is small compared with the crack dimensions, a linear elastic stress field may be assumed around the crack tip. In these situations, the behavior of fracture is controlled by the magnitude of the stress intensity factors K_I, K_{II}. These are the coefficient of $r^{-1/2}$ in the singular part of the expansion of the stress ahead of the crack as a function of r (the distance from the tip). Additionally, the stress intensity concept is important in terms of crack extension as critical values of the stress intensity factor (SIF) govern crack initiation[1]. The combined effect of modes I and II, under tensile and shear loading, presents difficulties in analysis. Such calculation may be carried out using finite elements and boundary element methods [3,4], or recently mesh free methods [5] and extended finite element methods [6]. For the modeling of crack extensions, the original crack length must be modified during the loading condition at each loading step. This requires calculating the displacement field near the crack tip at each load step and updating the crack length before the next load step is applied. This procedure is very difficult when using methods such as finite elements and boundary elements because this means updating the original mesh of the problem and this which demands time and effort consumption for the analyst.

This research presents a matlab procedure for the analysis of mixed mode fracture analysis using the approximate and analytical solution for crack problems available in the literature to compute and illustrate the stress intensity factors for different crack length without the need for re-meshing or further calculations. The new illustration is shown in the way of presenting the results in 3 dimensional form for the relations between the displacement and stresses versus, crack angles, and crack tip radii.

2-Governing Equations

The near crack tip stress field equations for mixed mode stress intensity factors in infinite plate are given in reference [2] as follows:

$$\sigma_x = \frac{K_I}{(2\pi r)^{1/2}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right).\sin\left(\frac{3\theta}{2}\right)\right] + \frac{K_{II}}{(2\pi r)^{1/2}}\sin\left(\frac{\theta}{2}\right)\left[2+\cos\left(\frac{\theta}{2}\right).\cos\left(\frac{3\theta}{2}\right)\right]$$

$$\sigma_y = \frac{K_I}{(2\pi r)^{1/2}}\cos\left(\frac{\theta}{2}\right)\left[1+\sin\left(\frac{\theta}{2}\right).\sin\left(\frac{3\theta}{2}\right)\right] - \frac{K_{II}}{(2\pi r)^{1/2}}\sin\left(\frac{\theta}{2}\right).\left[\cos\left(\frac{\theta}{2}\right).\cos\left(\frac{3\theta}{2}\right)\right]$$

$$\tau_{xy} = \frac{K_I}{(2\pi r)^{1/2}}\cos\left(\frac{\theta}{2}\right)\left[\sin\left(\frac{\theta}{2}\right).\cos\left(\frac{3\theta}{2}\right)\right] - \frac{K_{II}}{(2\pi r)^{1/2}}\cos\left(\frac{\theta}{2}\right).\left[1-\sin\left(\frac{\theta}{2}\right).\sin\left(\frac{3\theta}{2}\right)\right]$$

..(1)

The principal stresses can be calculated from the above components using Mohr circle formula given in Reference [7] as follows:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\end{Bmatrix} = \left(\frac{\sigma_x+\sigma_y}{2}\right) \pm \left[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + (\tau_{xy})^2\right]^{1/2}$$

$$\sigma_3 = \begin{Bmatrix}\upsilon(\sigma_1+\sigma_2).........Plane-strain\\ 0.......................plane-stress\end{Bmatrix}$$

...........(2)

The displacement field for the same modes are also given as follows:

$$u = \left(\frac{K_I}{4\mu}\right)\left(\frac{r}{2\pi}\right)^{1/2}\left[(2\kappa-1)\cos\left(\frac{\theta}{2}\right)-\cos\left(\frac{3\theta}{2}\right)\right] - \left(\frac{K_{II}}{4\mu}\right)\left(\frac{r}{2\pi}\right)^{1/2}\left[(2\kappa+3)\sin\left(\frac{\theta}{2}\right)+\sin\left(\frac{3\theta}{2}\right)\right]$$

$$v = \left(\frac{K_I}{4\mu}\right)\left(\frac{r}{2\pi}\right)^{1/2}\left[(2\kappa+1)\sin\left(\frac{\theta}{2}\right)-\sin\left(\frac{3\theta}{2}\right)\right] + \left(\frac{K_{II}}{4\mu}\right)\left(\frac{r}{2\pi}\right)^{1/2}\left[(2\kappa-3)\cos\left(\frac{\theta}{2}\right)+\cos\left(\frac{3\theta}{2}\right)\right]$$

............(3)

where,

$$\mu = \frac{E}{2(1+\upsilon)}$$

$$\kappa = \begin{Bmatrix}(3-\upsilon)/(1+\upsilon).........Plane-stress\\ (3-4\upsilon)..................Plane-strain\end{Bmatrix}$$

The Brown solution for K_I stress intensity factor of finite plate under tension load for central cracked plate (Reference [6], Page 38) , limited to half crack length to plate width ratio (a/W) = 0.6, is given in Reference [6] as follows:

$$K_I = C\sigma\sqrt{\pi a} \qquad \textbf{.............(4)}$$

Where the compliance function *C* is given as follows:

$$C = 1 + 0.256\left(\frac{a}{W}\right) - 1.152\left(\frac{a}{W}\right)^2 + 12.2\left(\frac{a}{W}\right)^3$$

σ is the applied tensile stress, *a* is half the crack length, and *W* is the plate width.

The K_{II} stress intensity factor is equal to 0.75 K_I as stated in Reference[3].

The elastic stress field in the vicinity of a crack tip, as given by equations (1), shows that as r tends to zero the stresses become infinite (i.e. stress singularity at the crack tip). Since many structural materials deform plastically above the yield stress, there will be in reality a plastic zone surrounding the crack tip, and the elastic solution for such situations, may require modification to some of the linear elastic fracture mechanics concepts. The two physically acceptable yield criteria for metals and alloys are the well-known Tresca and von Mises yield criteria. In this work the von Mises criterion will be considered which requires that the distortion energy per unit volume approaches its critical value. In simple tension this criterion can be expressed in terms of principal stresses
as follows [7]:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2Y^2 \quad \text{.....……..(5)}$$

where Y is the yield stress of material in simple tension.

The summation of the square terms in the above equation termed the equivalent stress (σ_e) which represents the strength of the plasticity near the crack tip and can be plotted in a contour form to show the boundary of the plastic zone surround the crack tip.

3- Analysis Procedure

The steps for the procedure of carrying out the mixed mode fracture analysis are shown in Figure (1). A matlab program is written based upon the mentioned steps to carry out the analysis. To validate the procedure and the program a case study of central cracked plate with different crack angles and loading are chosen for this purpose and the results are shown in the next section.

4- Results and Discussion

An Aluminum plate of dimension 50 mm x 20 mm and thickness 1 mm, with initial crack length of 6 mm, given in Reference [8], with modulus of elasticity 70 GN/m^2, and possion's ratio 0.33 and Yield stress of 17.5 MN/m^2 is considered according to Reference [9], (yield stress for Aluminum is 15-20 MPa).

The plate considered to demonstrate the analysis is a central cracked plate of width W and half crack length a. using the developed matlab program. The analysis starts by considering an a/W ratio of 0.1 and a 25 stress increments starting at a stress value which just cause yielding. The results for this analysis are as follows:

Figure (2) shows the behavior of the calculated stress intensity factors K_I, K_{II} for each load increment and crack length value. It is clear from Figure (2)-(b) and (c) that the K_I, K_{II} values increases nonlinearly with the increase of the load increment and the calculated crack length value.

Figure (3) illustrate the behavior of displacements with the crack angle and the crack tip radius for each stress increment starting with initial half crack length of 2. It is clear from this figure that the behavior of the displacements is somehow periodic with both crack angle θ and crack tip radius ***r***. Figures (4) and (5) presented the calculated Cartesian, principal and equivalent stresses and the yield stress for each crack length. These figures are very important because they show the behavior of the calculated stresses especially the equivalent stress with the given yield stress of the material which is an indication of the failure of the plate material.

Figure (6) shows the behavior of the calculated displacements for each crack angle and crack tip radius. This combined relation shows clearly the interaction between the crack angle and the crack tip radius for mixed mode fracture analysis. Figures (7) and (8) are the new illustration for the behavior of the Cartesian, the principal and the equivalent stresses for each crack angle and crack tip radius. It is clear that the behavior is not linear and the calculated values are behaving periodically with the increase of the crack angle and the crack tip radius. Finally, These figures are very important for designers since they described the behavior of the mixed mode fracture mechanics analysis in a new illustration which gives a clear picture for the relation between the different parameters presented in this research.

5-Conclusions

From the above analysis, the following conclusions can be drawn:

1- The new illustration of results for mixed mode fracture analysis presented in this paper shown to be very convenient and essential for describing the behavior of stress intensity factors, and stress states at different crack angle and crack tip radius.

2- The developed procedure reduced the need for sophisticated numerical analyses such as finite elements or boundary elements, which require more time and effort, to calculate the same parameters tackled in this research.

5- References

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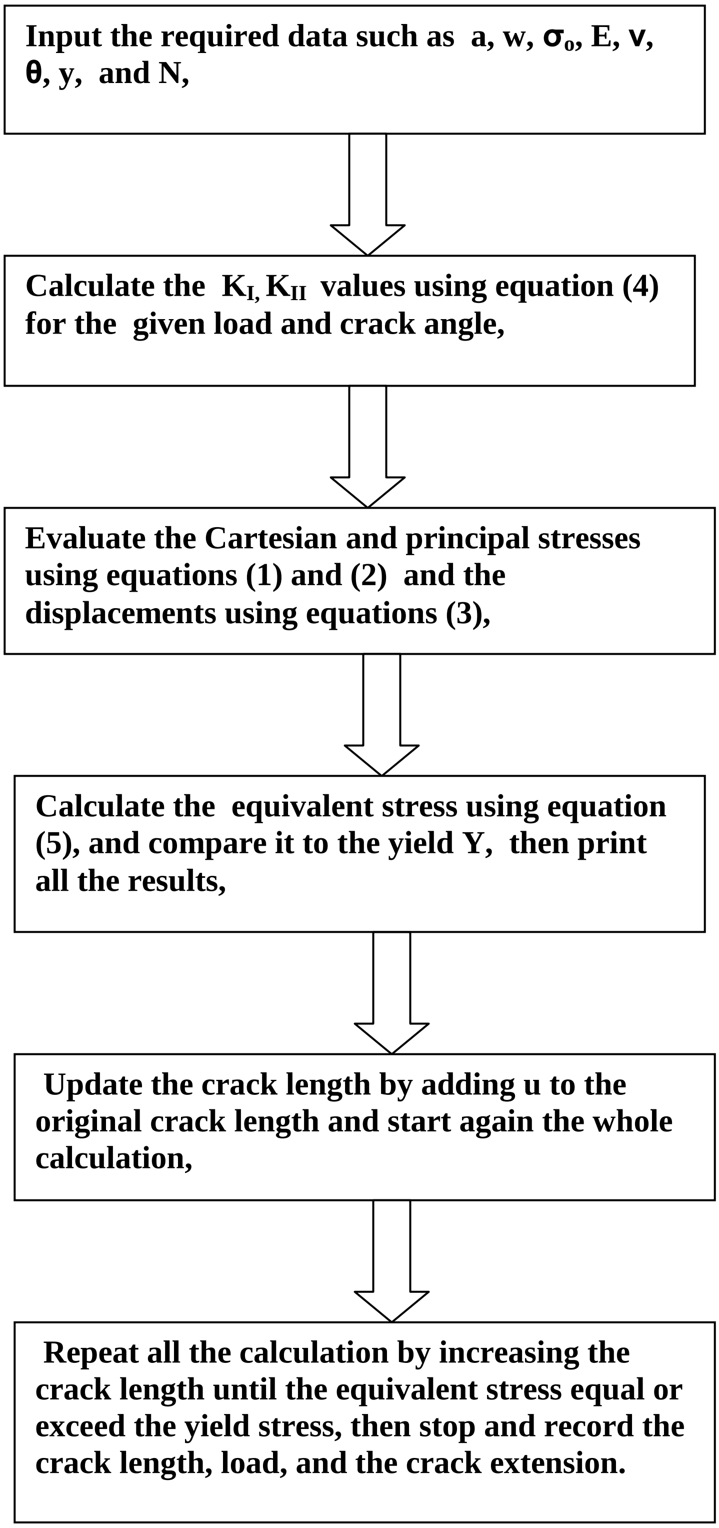

Fig. (1) Procedure for mixed mode fracture analysis using matlab.

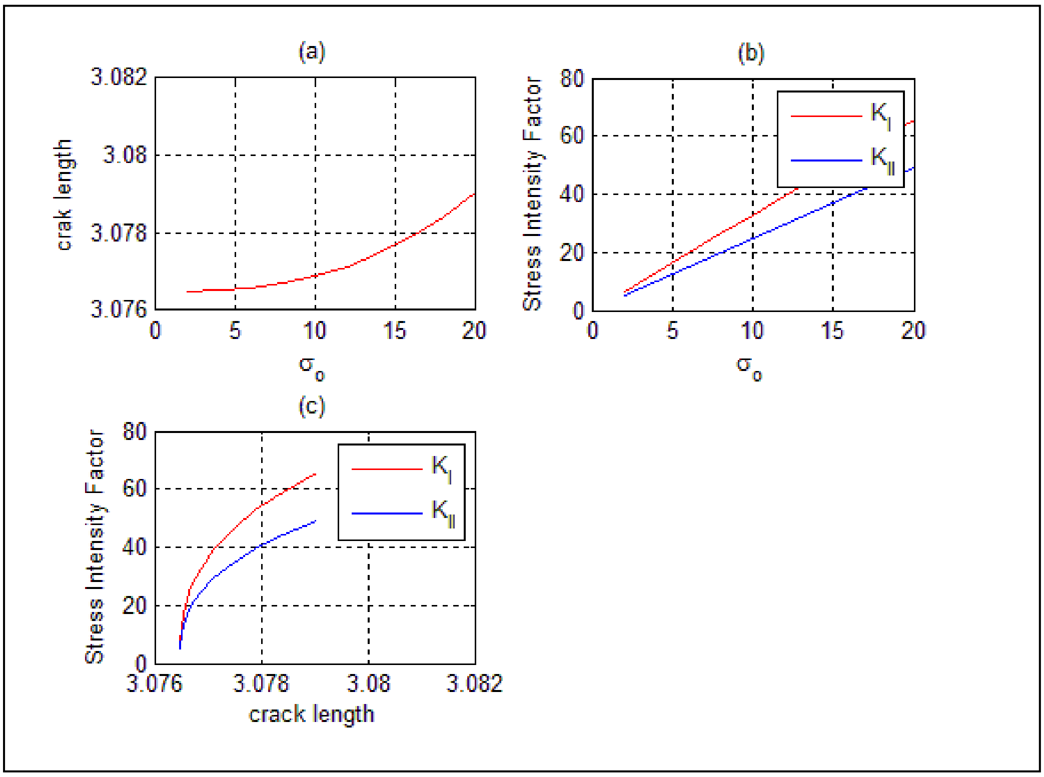

Fig.(2) The behavior of Stress intensity factors against applied load and crack length .

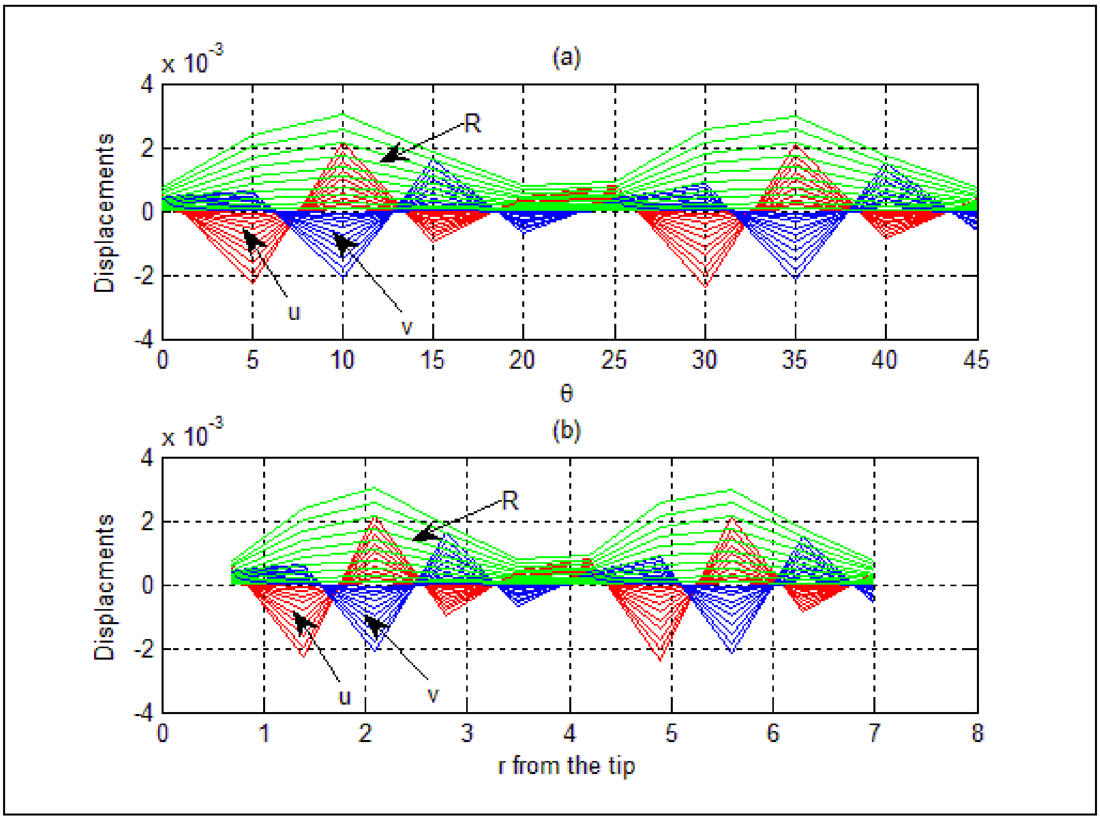

Fig.(3) The behavior of displacements for each crack angle and tip crack tip radius.

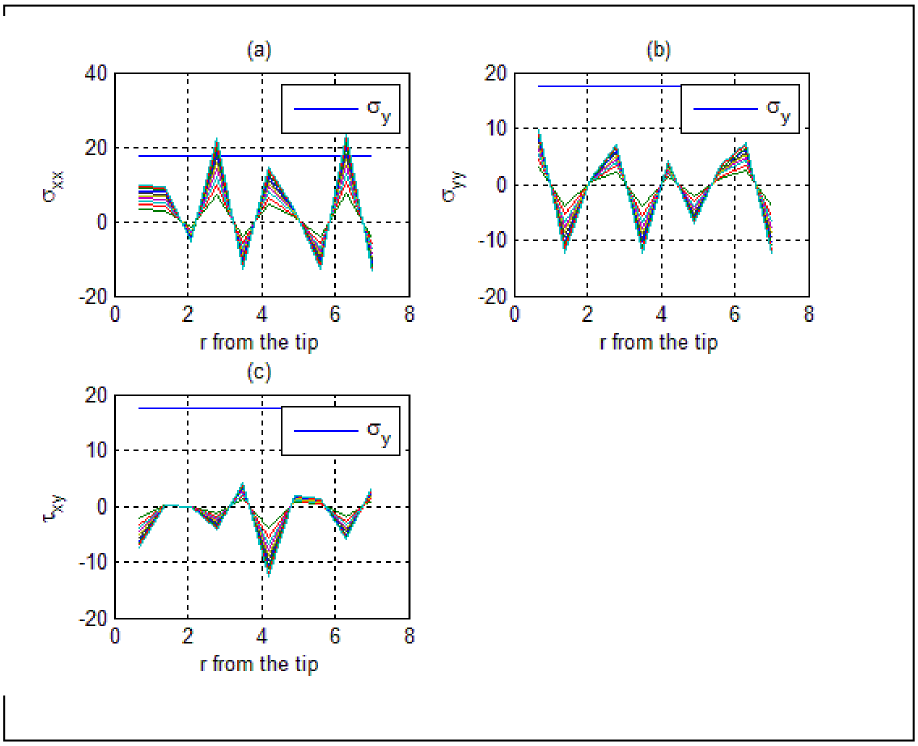

Fig.(4) Relationship between Cartesian stress components and crack tip radius for each crack angle.

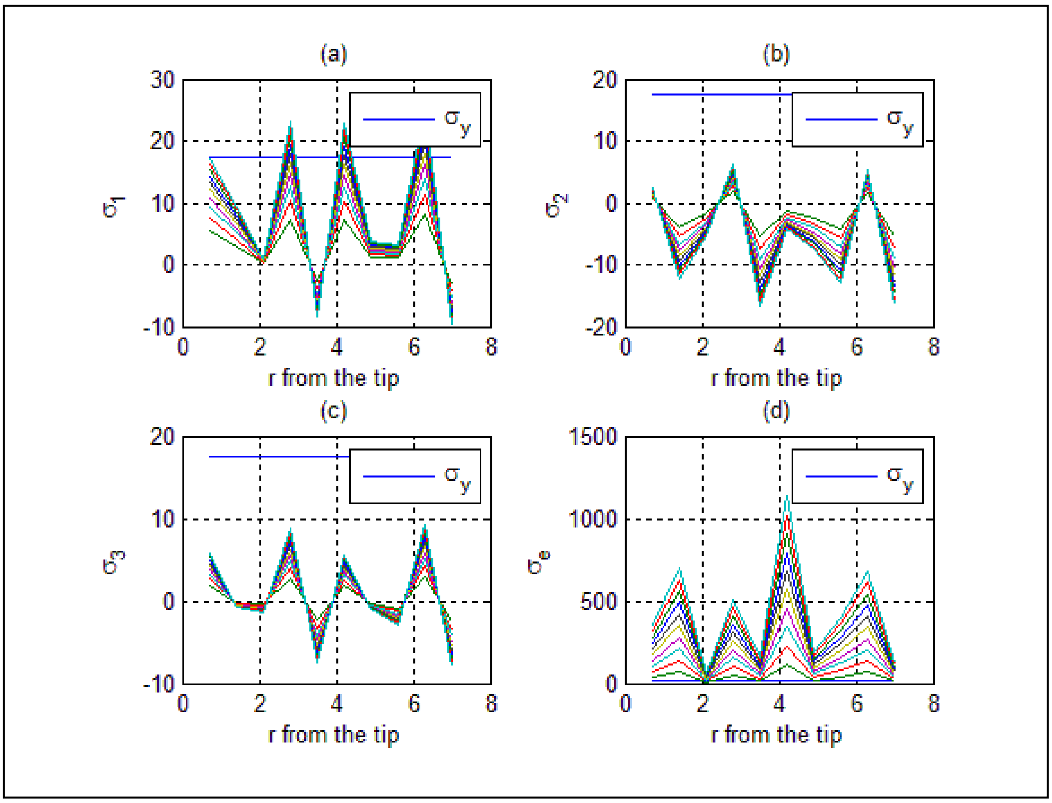

Fig.(5) Relationship between principal stresses and crack tip radius for each crack angle.

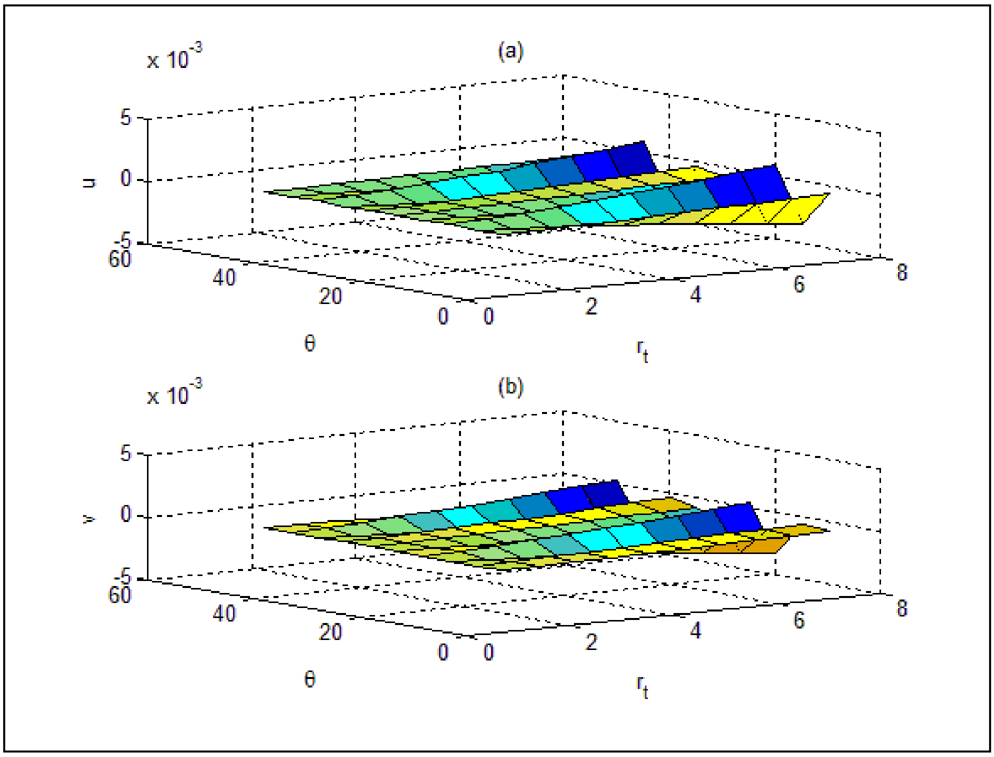

Fig.(6) The calculated displacements for each crack angle and crack tip radius.

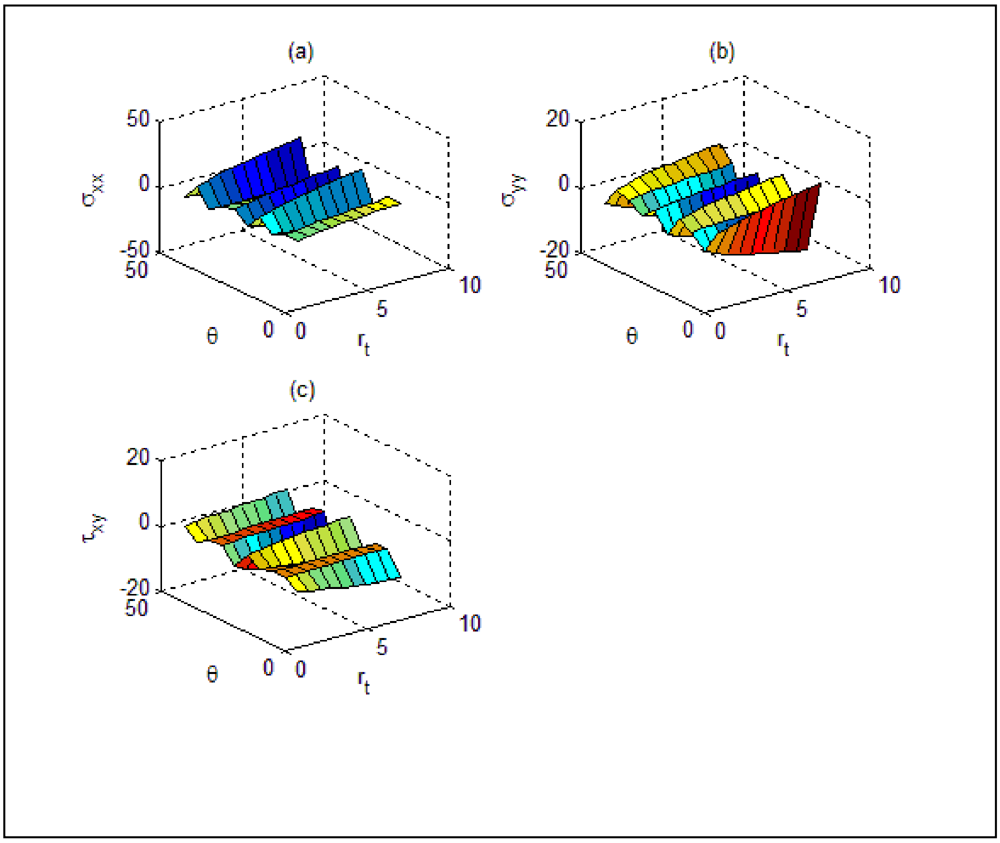

Fig.(7) The calculated Cartesian stresses for each crack angle and crack tip radius.

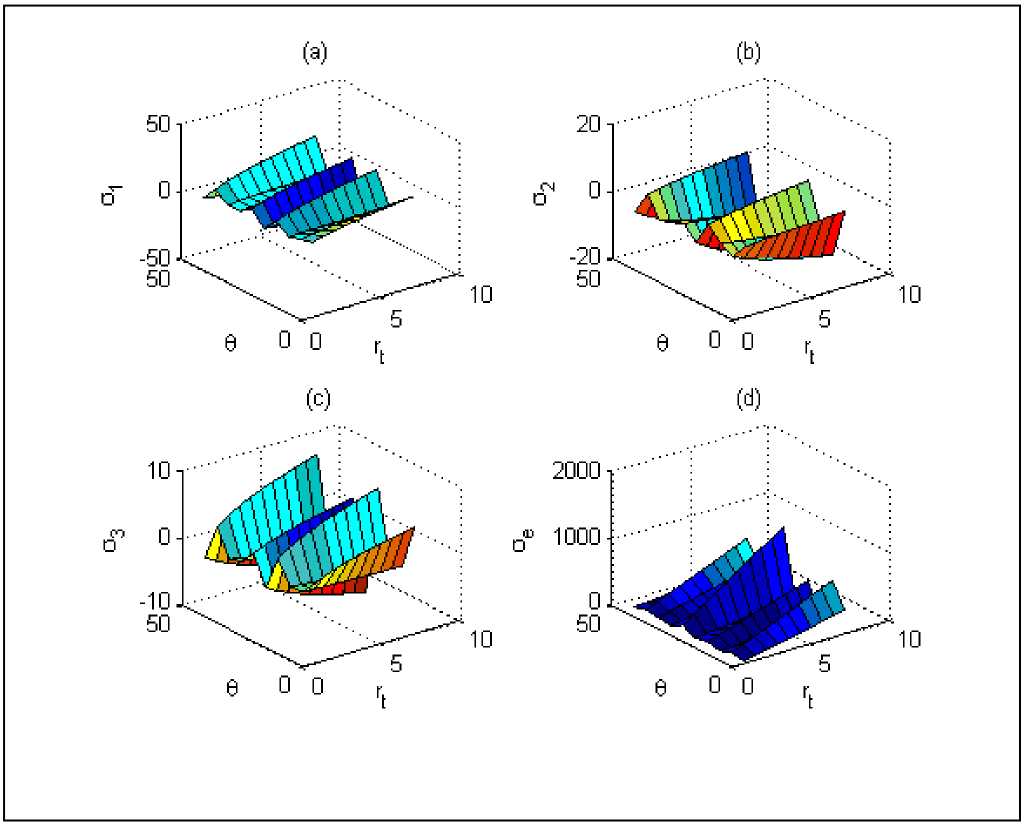

Fig.(8) The calculated principal stresses and equivalent stress for each crack angle and crack tip radius.